The Dualism of Learning Study: Understanding and Practices of Student-Centered Learning in Mathematics Lessons

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Abstract
There have been ideological variations in understanding, focus and practices of student-centred learning (SCL) across the world. This study explored the teachers' ways of understanding and practising SCL in enhancing student learning of mathematics concepts. Using interviews, lesson preparatory meetings, reflective journals, and students' tests, the study employed a phenomenographic variation framework, coding strategies and paired sample t-test to analyse data. The results show that when teachers share their experiences on SCL, they come up with a wider understanding of the best SCL pedagogical strategies. They unfold the SCL teaching challenges, especially in overcrowded classrooms. As such, teachers changed their ways of understanding (experiencing) SCL from seeing it as a methodological orientation to subject content and object of learning. The more teachers were engaged in the learning study, the better their pedagogical teaching practices became focused on students learning complex mathematical topics.

Keywords: *Learning study, Student learning, Student-centred learning, Teacher learning, Variation theory.*

INTRODUCTION

There are evident variations in understanding, focus and implementation of Student-Centred Learning (SCL) across the world. The literature portrays these variations are categorized around conceiving SCL in terms of methodological orientation - a way of according to students' opportunity to meet their learning needs, choose what to learn, and how to be assessed (O'Neill & McMahon, 2005); a way of shifting power and responsibility from the teacher to students (Mushi, 2004); and the act of engaging learners in experiencing critical aspects of the object of learning (Msonde & Msonde, 2017a; 2019). These differences originate mostly from diverse theoretical underpinnings, not from teachers' practical experiences.

Like many other countries in the world, Tanzania introduced SCL in school curriculum reform, which emphasized the use of participatory methods to shift teachers from teaching to learning (Wangereja, 2003). Besides being trained on the ways to implement SCL in their pre-service and/or in-service teacher professional development (TPD), teachers in Tanzanian schools have not changed their pedagogical practices. These teachers employ transmittal teaching strategies in teachers' colleges and secondary and primary schools. Among the factors that hinder teachers from effective implementation of SCL include prevailing constraints in schools such as large classes, inadequate resources, and teacher overload. Though acknowledging these realities, Msonde and Msonde (2018) argue that the school teachers were not well prepared in teachers' colleges and/or in-service orientations to implement SCL innovations. Indeed, they relied on one way of seeing SCL as a codified participatory method.

Little has been done to explore teachers' practical experiences of SCL in overcoming formidable classroom challenges in Tanzania. The underlying assumption of this study was that when teachers come together and share their experiences on SCL practically, they could come up with a wider understanding of the best way to implement SCL. Additionally, it could unfold the teachers' challenges in understanding and practising SCL. Thus, exploring school teachers' ways of experiencing and practising SCL in promoting student learning is essential to realize the teachers' practical knowledge and their learning trajectory.

Theoretical context

A learning study is an "action research which aims to improve classroom teaching and learning by enhancing teacher professional development" (Keung, 2009: 33). In line with Marton and Lo (2007), it is a designed experiment aiming at making students as well as teachers' learning possible. In this process, teachers as practitioners learn from their practices which results in improving students' learning outcomes (Pang, 2006). An effective learning study improves not only students' learning but also high-quality teacher professional development (Msonde & Msonde, 2017b).

On the one hand, a learning study is grounded in the theory of variation (Pang, 2006) whereby teachers practice their lessons focusing on specific critical aspects of an object of learning aiming at achieving student capabilities. On the other hand, the theory of variation was built from the phenomenography perspective that different people conceive the same phenomenon differently (Marton & Booth, 1997). These differences are due to the variations among individuals in experiencing a phenomenon (Marton & Tsui, 2004). To experience a phenomenon, one has to focus on the critical aspects of the phenomenon simultaneously (Lo *et al*., 2005). However, discerning critical aspects is not possible without one to experience variations on those critical aspects. In this context, learning becomes a function of discernment, variation and simultaneity. Thus, student learning achievement depends on the way teachers handle the object of learning in classroom practices (Msonde & Msonde, 2018). As such, this study explored two research questions, first, how do Mathematics teachers understand and practise SCL in their teaching practice? Secondly, how did participation in the learning study improve teachers' and student learning?

METHODOLOGY

Setting and participants

This study was conducted at one community secondary school in Tanzania. The secondary school was purposively selected following the willingness of the school administrator to host the study and the school Mathematics teachers' acceptance to participate in this research. The Tanzania secondary education system resembles that of secondary education in the UK. However, the learning environment in Tanzania's secondary schools especially community-based ones, is associated with large classes, inadequate teaching and learning resources and teachers' heavy workloads (Athour, 2017). At the time of conducting this study, the school had 10 teachers. Three of them taught Mathematics subject. For confidentiality and ethical reasons, the researchers assigned pseudonyms (Teachers A, B, and C) to the three Mathematics teachers who later formed a learning study group.

Before the professional development (TPD), teachers were unaware of the learning study and the variation theory. Rather, they were trained by teachers training colleges to follow a particular method as prescribed in the curriculum. The methods prescribed in the curriculum focused on classroom pedagogical arrangement to bring about students' participation in the instruction and learning process. This study explored how the three Mathematics teachers understand and practise SCL to bring about student learning.

Study approach

This study was performed in two major phases to explore the way teachers experienced and practiced SCL before and during the learning study intervention. The teachers were oriented for two days before the learning study intervention so that they could learn about the theoretical underpinnings of the learning study. Thereafter, they were engaged in three learning study cycles (research lessons) to share their experiences with the way they understood and practiced SCL. Each research lesson included 3 pre-Lesson Preparation Meetings (LPMs). The first meeting (LPM1) involved teachers in selecting the object of learning to address. They also shared their experiences with challenges or difficulties that students faced in learning what was to be taught and designed the pretest. The second meeting (LPM2) was aimed at reflecting on the pretest students' learning outcomes. They found that there were critical aspects for students learning what was taught. Finally, in the third meeting (LPM3), teachers planned the lesson (the intended object of learning). There were also 3 post lesson preparation meetings. These meetings reflected the way the lesson was conducted in the three research lessons. The teachers also filled in the reflective journal (TRJ) at the end of each research lesson.

Data collection

Data were obtained from interviews, supplemented by teachers' lesson preparatory meetings and reflective journals. The teachers were interviewed before and immediately after the orientation as well as at the end of each research lesson. Each interview was audio-recorded and lasted for approximately 40 minutes. The interview was intended to explore the teachers' practical reactions and understanding of SCL before and during an intervention. The lesson preparatory meetings provided important data on how teachers enacted the SCL in classroom settings. In addition, the teachers' reflective journal provided supplemental data which revealed the practical experiences and challenges in implementing SCL in their school milieu.

Data analysis

Analysis of interviews, teachers' preparatory meeting lessons, and reflective journals followed phenomenographic analytical conventions (Akerlind, 2008; Pang, 2006). These data were further coded by the first coder using ATLAS. Ti software was used to generate a set of main and sub-codes. During the coding process, the transcripts were studied holistically to capture their general meaning to recognise important parts that were marked as significant quotes. Then, the first coder was compared with coded segments to identify critical aspects that were core to realise the teacher's understanding of SCL practically at different time points. The coder established the set of core themes (main codes) and consistently improved the code description for more than three rounds. To ensure the trustworthiness of the coding scheme, the codes were given to the second coder for member checking. The coders agreed to 12 out of the 17 codes. Their level of agreement was 0.82 (Cohen's κ), which exhibits excellent inter-coder agreement. The discrepancies between them were resolved through negotiation. Then, the first coder refined the codes and re-coded the data using the new coding scheme which comprised 5 themes: teacher learning, teaching methods, student engagement, critical aspects and the object of learning.

FINDINGS

The results of this research are presented in several subsections. First, we present the teacher's views on their learning and SCL practices before the intervention, followed by their views and practices on it during the intervention.

Experiencing and practising SCL before learning studies

Teachers in the learning study group were asked through in-depth interviews how they understood and practised SCL in classroom settings. The teachers had similar views, and one of them had that to say:

> To me, SCL is to implement participatory methods that make learners participate fully in the entire teaching and learning process. To practise learner-centred teaching you need to apply participatory instructional methods such as group discussion, role play, simulation, and others. And, this is different from lecturing in which students mainly become listeners (Teacher B).

When teachers were asked further to explain what they normally focus on in planning their SCL lesson, Teacher C noted that:

> The first thing I do is to look at the school syllabus which shows alternative teaching methods for various topics. Then, I select the methods that enable students to participate fully in learning the topic in question. You know, although the methods are there, I need to think about what I will be doing in the lesson and how my students will be fully involved in the learning process (Teacher C).

It seems that the teachers experienced SCL as a *methodological classroom orientation*. Teachers put much emphasis on particular methods during lesson preparation and teaching so that the students become active in the learning process. It was likely that the issue of "what is taught" was not emphasized. However, despite being trained to implement participatory methods, the teachers adopted lecturing teaching strategies due to the overcrowded nature of classes and scarcity of teaching and learning resource constraints. Teacher A had that to say:

> I am among the teachers who practise participatory methods less. And this is because of the large number of students in the class. It is difficult to use SCL when you have large classes. For example, in one of my classes, I have an average of 80 students. You find even the space to pass through checking the students' learning progress during instruction becomes difficult to obtain (Teacher A).

Although the teachers understood the need to practise the participatory instructional method in the SCL teachings, they found its implementation was impractical in the prevailing classroom realities. It seems the teachers had inadequate innovative knowledge to implement SCL in such school challenges.

Experiencing and practicing SCL during research lesson 1

The intended object of learning

The teachers selected the object of learning, "relationship between sides of a right triangle and trigonometric ratios". They agreed on developing students' computation skills in trigonometric ratios. They designed and administered the pre-test to explore student experiences on trigonometric ratios to Form Two students. Overall, the class scored an average of 10.5% (see Table 1). During the reflection on students' pre-test results, Teachers came up with four critical aspects for students to experience the sides of the right triangle and trigonometric ratios. These were directional, perpendicular, length and relational sides. Three levels of structuring the lesson were identified: understanding the sides of a right triangle, computing and applying trigonometric ratios. Teacher C had that to say:

> We have to divide the lesson into three levels. We start with developing students' understanding of the sides of the right triangle: opposite, adjacent, and hypotenuse. Here we focus on directional, perpendicular, and longest sides. Second, we focus on developing student computation skills of sine, cosine, and tangent of an angle. That is, we focus on the sides' ratios or the relation of sides. The third is to develop students' ability to use sine, cosine, and tangent in estimating/finding the height of the wall/tree and the width of the river. We have to consider a variety of teaching methods and follow our pedagogical structure. But, what is important should be how our students are engaged in those aspects one after the other as we planned. They should work in pair groups and share their answers with other groups (Teacher C).

In the end, the teachers designed the lesson plan for this object of learning. They set three capabilities to be developed. These were understanding opposite, adjacent and hypotenuse sides; computing the sine, cosine, and tangent of an angle; and applying trigonometric ratios to estimate height, width, and length in various scenarios. However, the teachers did not set up patterns of variation in this lesson, particularly in those critical aspects they identified.

The enacted object of learning

Teacher A taught this lesson in class 2A. The lesson was divided into three stages: guiding students to experience sides of right triangles (opposite, adjacent and hypotenuse); computing trigonometric ratios (sine, cosine, and tangent); and applying trigonometric ratios (length, height and width). In the first stage, Teacher A labelled the right triangle sides **a**, **b** and **c**; and particular acute angle **θ** (see figure 1). He required the students to point out the opposite, adjacent, and hypotenuse sides of angle **θ**, but they failed. By using Figure 1a, Teacher A explained how the opposite, adjacent, and hypotenuse sides are obtained.

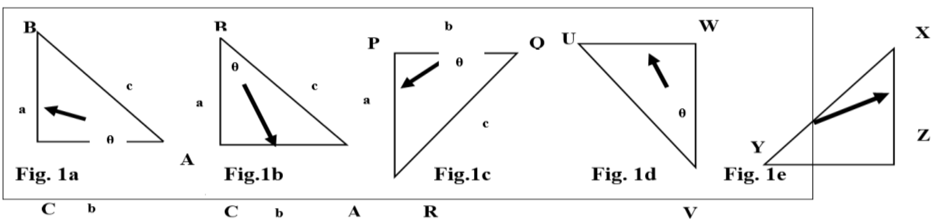

Figure 1: *The right triangle figures*
Source: Author creation

He also did the same by shifting the position of the acute angle of interest using Figure 1b; and changed the triangle orientation in Figure 1c. In those examples, he tried to vary the acute angle **θ** position as well as triangle orientation separately; to make students learn how opposite, adjacent, and hypotenuse sides shifted accordingly. Later on, he emphasized his explanations by using various right triangle figures in different orientations (see figures 1c, 1d &1e).

In the second stage, Teacher A guided students to unfold the trigonometric ratios formula by computing six ratios of sides in a right triangle concerning an acute angle 30^0. He provided students with a paper sheet that had 3 similar right triangle diagrams. These diagrams were designed by the learning study group. Each group dealt with only one right triangle. These triangles differed in size but the students were told to focus on an angle (30^0). From their results, he asked the students to identify three important ratios that relate to sine 30, cosine 30, and tangent 30 as stipulated in mathematical tables. Such practices enabled the students to come up with a workable formula SOTOCA/HAH presented below.

$$\boldsymbol{Sine\theta} = \frac{\text{Opposite}}{\text{Hypotenuse}};\ \boldsymbol{Tangent\theta} = \frac{\text{Opposite}}{\text{Adjacent}};\ \boldsymbol{Cosine\theta} = \frac{\text{Adjacent}}{\text{Hypotenuse}}.$$

Later on, Teacher A guided students in using the formula through many examples. In the third stage, he guided the students to perform an application question that estimated the height of a tree to exemplify how tangent could be used. However, Teacher A was unable to provide examples of applications of sine and cosine. To create patterns of variation, he could have designed three different examples that require students to estimate the height of a tree; the width of the river; and the length of a ladder. In this way, students could have experienced uses of tangent, sine, and cosine simultaneously, and in a powerful way.

The lived object of learning

To explore further the manner students experienced sides of right triangles and trigonometric ratios a post-test (which was parallel to the pre-test) was administered. The t-test paired sample analysis was applied using the SPSS 16.0 version to compare the mean of pre and post-tests. The results showed the differences in students' performance between the two tests were statistically significant ($p<0.05$). Students had an overall mean score of 10.5% and 42.6% in pre and post-tests respectively, which was a gain of 32 points as shown in Table 1.

Table 1: *Students learning outcomes in Lesson 1*

Lesson 1		*Mean*	*N*	*Std. Deviation*	*Std. Error Mean*	*t*	*Sig.*
Pair 1	*PRETEST1*	*10.588*	*85*	*13.8033*	*1.4972*		
	POSTTEST1	*42.694*	*85*	*25.2006*	*2.7334*	*-14.237*	*.000*

The researchers asked Teacher A to reflect on the lesson. Teacher A's responses seemed to focus much on the manner students were engaged *(student engagement)* in the content manipulations. That is a derivation and use of the mathematical convention SOTOCA/HAH.

> I think the lesson was good in the sense that students were engaged well in learning the sides of right triangles, opposite, adjacent, and hypotenuse sides. You see, they were able to come up with important sides' ratios themselves, which formed a SOTOCA/HAH formula, which they used to compute sine, cosine, and tangent. When they were through with those, they used this knowledge to estimate the height of trees and the width of rivers. In all stages, I was interested in enabling students to achieve what was intended. (Teacher A).

Moreover, Teacher A believed that his lesson was SLC-oriented because it involved students in important components of what was taught sequentially.

Experiencing and practicing SCL during research lesson 2

The intended object of learning

The learning study group decided to deal with a slope of straight lines. The slope was seen as among the problematic mathematical concepts that hamper student learning. Teacher C described some difficulties students faced in learning linear slope. These included inadequate manipulation skills and interpretation of slope as a rate of change. Therefore, the teachers administered a pre-test to explore students' prior experiences on slopes. Simple descriptive statistical analysis was done and obtained a mean score of 12.5% indicating poor performance. Based on student performance, Teacher B argued that students were not aware of the slope concept and lacked mathematical computation skills. Table 2 shows the four stages of the lesson plan and the identified critical aspects namely the angle of inclination, and vertical and horizontal distances that students should discern separately and sequentially.

Table 2: *The intended object of Learning Lesson 2*

Stage	Teachers intended activities	Vertical distance	Horizontal distance	The angle of inclination/ Slope
1	Design examples that vary the vertical distance while horizontal distance remains invariant. Guide students to experience slope in that situation.	V	I	V
2	Design examples and guide students to see the slope in the situation where horizontal distance varies while vertical distance is invariant.	I	V	V
3	Design an example and guide students to learn slope in a situation where that angle of inclination varies while either vertical or horizontal remains invariant.	V/I	I/V	V
4	Design examples and guide students to experience slope when all critical aspects vary (use teaching tool E_4).	V	V	V

The teachers had the view that the students did not discern vertical and horizontal distances as well as angles of inclination. As such, Teacher C suggested using real scenarios such as mountains/hills to familiarize students with the concept of slope.

The enacted object of learning

Teacher C taught this lesson and arranged the students in groups before his teaching. The lesson was divided into four main scenarios. In the first scenario, Teacher C asked students to volunteer to draw a graph on the board, locate points O(2,3), A(6,8), B(6,6), C(6,4) and, join three lines from point O to the rest. These straight lines varied in both angle of inclination and vertical distances but had invariant horizontal distances. He asked the students to describe why there were differences in slope. Responses from the students revealed that *height* (vertical distances) was a determinant factor of steepness differences. Although the angle of inclination (DOA >DOB> DOC) differed among the three lines, the students did not experience it at this point and it was taken for granted. Teacher C realized that the students did not consider the angle of inclination as a determinant of slope variation. He used a mountainous diagram (see Figure 2) to show two sides A and B, which varied in steepness in his second stage. Teacher C required students to reason out factors influencing steepness differences in two hilly sides A and B.

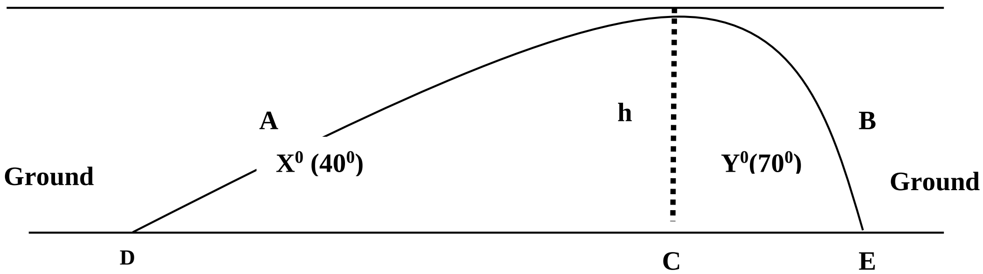

Figure 2: *Hilly diagram with sides A and B*
Source: Author creation

The students were able to identify the angle of inclination as another determinant of steepness variations but were unable to determine the horizontal distance, which, in this example, also varied. In the third stage, Teacher C skilfully employed another example to enable students to determine the horizontal distance as the determinant of the steepness variation of straight lines in the x/y plane. Students experienced this reality after being guided to draw lines starting from point O (0,0) to points A (4, 4), B (2, 4), and C (8, 4) in the x/y plane. The lines varied in horizontal distance but remained invariant in vertical distance. In stage 4, he asked the students in pairs to explore the vertical and horizontal units of each scenario so that they could discern the various horizontal critical aspects. Moreover, Teacher C used another example, as illustrated in Figures 3a to 3f, to enable students to discern the horizontal and vertical distances sequentially to understand variations in slopes. Teacher C did not use this opportunity to guide students in discerning all aspects of those figures simultaneously, as was expected. Instead, he focused mainly on Figure 3 (g, h, I, j, k, & l), which was intended to relate those diagrams with slopes in the students' environment.

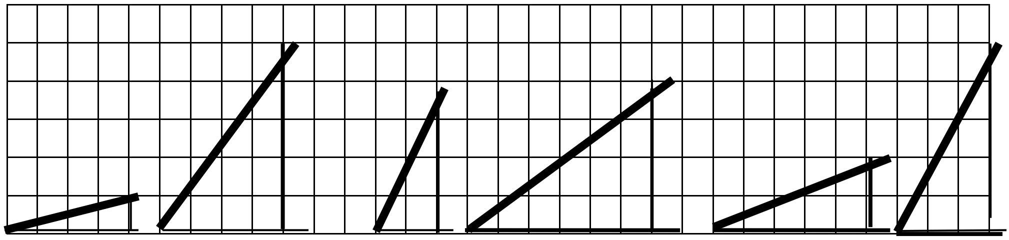

Fig. 3a Fig. 3b Fig. 3c Fig.3d Fig. 3e Fig. 3f

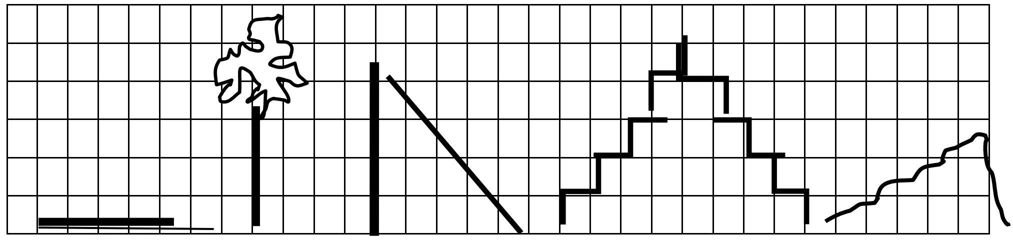

Fig. 3g (floor) Fig. 3h (tree) Fig.3i(wall) Fig. 3j (ladder) Fig. 3k(staircases) Fig. 3l(hill)

Figure 3: *The inclined scenarios in the x/y plane*
Source: Author creation

Then, he guided the students to experience slope as a ratio between vertical and horizontal distances, using line joining points (5, 2) and (8, 1) as an example. The students explore the vertical and horizontal distances of that line as well as its ratio. They were guided to derive by themselves the slope formula presented below.

$$\textbf{Slope} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}; \quad \textbf{Slope} = \frac{\text{Y1} - \text{Y2}}{\text{X1} - \text{X2}}$$

Teacher C did not use this opportunity to guide students in discerning all aspects simultaneously as was expected. For example, figures 3(a, b, c, d, e & f) were not taken into account. These figures were intended to enable students to discern both the vertical and horizontal distance aspects simultaneously.

The lived object of learning

After the lesson, students did a post-test, which was parallel to the pre-test. The result from the t-test paired sample analysis (see Table 3) showed that the differences in students' performance between the two tests were statistically significant ($P<0.05$).

Table 3: *Students learning outcomes in Lesson 2*

	Lesson 2	Mean	N	Std. Deviation	Std. Error Mean	T	Sig.
Pair 1	PRETEST2	12.471	85	6.6826	.7248	-16.547	.000
	POSTTEST2	59.024	85	27.8721	3.0232		

The researchers asked Teacher C to describe how he used the variation theory in his lesson and had to say,

> In my lesson, I guided students to draw three straight lines from a single point to different points, which differed in vertical distances and angle of inclination. However, those lines had the same horizontal distances. Here I wanted my students to focus on the vertical distance aspect. I also used a hilly figure diagram with two unequal slope sides. The vertical distances of the hill on each side were equal, but they varied in horizontal distances. I wanted students to focus on the horizontal distance aspect. I also planned an example similar to example 1 to make students focus on angle differences because, in the first example, the inclined angles also varied. This process made students experience how those aspects influence slope (Teacher C).

It seems Teacher C experienced SCL as teaching that *engages students* in experiencing critical aspects of what is taught. The role of the teacher in the process was to identify and organise critical aspects of the object of learning. In this lesson, the critical aspects identified include angle of inclination, and vertical, and horizontal distances and organized them in such a way that students attend to those aspects. This line of argument was also in line with teacher C's reflective journal.

Sharing with my fellow teachers helped me a little bit to improve my classroom teaching. Previously I provided mathematical formulas, demonstrated how to use them and provided students with some mathematical problems to solve... Now, I focus much on engaging them in various critical aspects they are supposed to learn in particular content. However, I still found it difficult to design examples that involve students in all critical aspects at the same time

Teacher C learned *(teacher learning)* from other teachers the way to improve his teaching. He was able to engage students in experiencing all critical aspects (vertical and horizontal distances) sequentially. That is, make each of the aspects vary while keeping invariant others for students to focus on the varied aspects, one to the next. However, he failed to use opportunities apparent in his lesson to engage students in experiencing those aspects simultaneously. As such, it was difficult for students to experience the impact of all critical aspects (vertical and horizontal distances) on the slope at the same time. This implies that despite his improvement, Teacher C was not conversant enough on the use of the variation theory, especially in the area of structuring dimensions of variation.

Experiencing and practicing SCL during research lesson 3

The intended object of learning

At the first meeting, teachers selected the topic of circles from the Form III syllabus. The "determinants of arc length of circular objects" were the object of learning. Teacher B saw this topic as important because many students failed to differentiate circles with spheres and they were confused with the concept of central angle with 360^0. The teachers administered the pre-test to explore students' prior experiences on this object of learning, whereby the students scored the mean mark of 25.6%. Based on the student's responses to some of the questions, it was observed that students lacked an understanding of the influence of central angle and radius on arc length. Therefore, the teachers identified two critical aspects: a change in central angle and a change in radius. Table 4 shows the stages of involving learners in experiencing determinants of arc length.

Table 4: *Summary of the intended object of learning of lesson 3*

Stage	Teachers intended activities	Central angle	radius	Arc length
1	Guide students to draw a pair of equal circles (in size), insert two radii that subtend equal central angle, measure the resultant arc length and discuss the results.	I	I	I
2	Guide students to draw a pair of equal circles (in size), insert two radii that subtend unequal central angles, measure the resultant arc length and discuss the results.	V	I	V
4	Guide students to draw a pair of unequal circles (in size), insert two radii that subtend equal central angle, measure the resultant arc length and discuss the results.	I	V	V
4	Guide students to draw a pair of unequal circles (in size), insert two radii that subtend unequal central angles, measure the resultant arc length and discuss the results.	V	V	V

The enacted object of learning

In the first case, Teacher B required students in their pair groups to draw two equal circles and from them draw two radii that subtended equal central angles of their interest. Then, they were required to measure arc length by using rulers and threads. The students were fully involved in drawing, measuring and presenting results. The aim was to enable students to experience the magnitude of arc distances when subtended by the same central angles in two equal circles (all aspects were kept invariant). This was expected to create a contrast to the next cases, in which those aspects varied in different situations.

In the second case, the teacher required the students to draw two equal circles. From each of them, they had to draw two radii (AB and CD) that subtended different central angles by varying central angles while keeping the radius invariant. They were further required to measure the resultant arc length in each of the circles and provide the results they obtained. The aim was to enable students to discern the central angle as the determinant of the arc length of circular objects. The students drew circles and measured radii, angles as well and resultant arc lengths in the two circles. Teacher B encouraged as many results as possible and wrote them on the board enabling students to learn from other groups. Subsequently, the teacher required the students to draw two circles of different sizes in their respective pair groups. He asked them to place two equal central angles in each circle and try to measure their resultant arc lengths. Contrary to previous cases, the teacher made students experience arc length in a situation where radii varied while central angles remained invariant. As he did in the previous case, he organized student results on the board and students studied them carefully. The results enabled students to experience that the arc distance was directly proportional to the radius.

In the fourth task the teacher required students to draw two circles of different sizes and from them measure different central angles in each circle. The students in their respective groups drew circles of different sizes with varied central angles (see Figure 4), measured their respective arc lengths, and presented their groups' results. He intended to enable students to experience the arc length in a situation where the radius and central angle vary simultaneously.

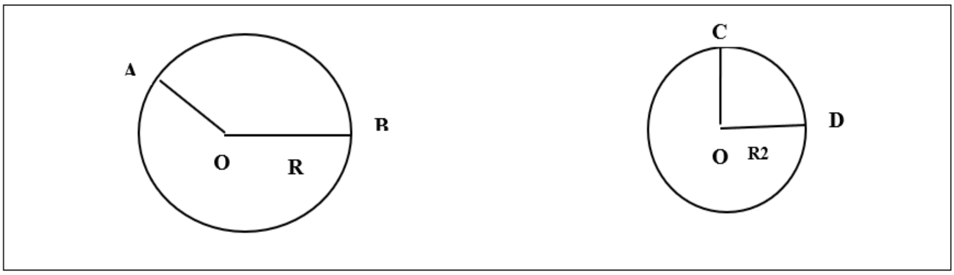

Figure 4: *Circles with varied central angles*
Source: Author creation

Teacher B organized the data into two groups and required students to think of those results and explain what they learnt from them. To develop mathematical skills among the students, he engaged them to derive the arc length formula and contextualized the use of the formula to students' real environment. For example, he guided the students to think of various circular objects in their scenarios and try to figure out how the arc length formula may be used in obtaining the distance of arcs around the circular objects. Students reflected on arcs of roundabouts where cars move to their side to avoid collisions with others. They also think of arcs of bicycle tyres formed between two spokes from the central bicycle hub. The teacher also required students to brainstorm which car between the taxi (small wheel size) and a bus (large wheel size) may cover a longer distance than the other provided that they move at a constant speed.

In a nutshell, Teacher B successfully involved students to attend to central angle and radius aspects in experiencing the arc length of circular objects separately and simultaneously. In each case, the teacher allowed the students to learn from others' group work results. They experienced diverse data, which enabled them to discern radius and central angle as determinants of arc length in a powerful way. The mathematical conventions developed thereafter improved students' potentialities in mathematical manipulative skills, reasoning, and applications. His ways of contextualizing mathematical problems using real examples were instrumental in enhancing student conceptual learning.

The lived object of learning

The post-test, which was parallel to the pre-test, was administered and the t-test paired sample analysis was performed using SPSS 16.0. The resultant data in Table 5 show that there were statistically significant differences ($p<0.05$) in students' performance between the pre-post-tests with an overall mean score of 24.6% and 56.8% respectively.

Table 5: *Students' learning outcomes in lesson 3*

Lesson	3	Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST	24.64	80	15.810	1.768	-14.687	.000
	POSTTEST	56.86	80	26.917	3.009		

The post-lesson interview was done and Teacher B filled in a reflective journal. The researchers asked Teacher B to explain how his lesson was SCL. The responses showed that Teacher B conceived SCL as engaging learners in attending to *critical aspects* of the *object of learning*.

> As I now know, SCL is a way of enabling students to experience critical aspects of what is taught. So, what I did was to ensure these critical aspects are well attained by students. In my lesson, I started with two circles with the same radius and central angle. I asked students to measure the resultant arc length of those circles, and they obtained the same arc length. Then, I varied the central angles of two circles while keeping the radius the same. When they measured the arcs they found a different length. I asked students why this was the case, and they pointed out confidently that it was because of differences in central angles. Later, I

> varied the radii while keeping the central angle constant. They found that the circle with a big radius had a longer arc length than that of the small radius, even though, both had the same central angle. I asked students why exist these differences, some said it was because circles differed in size; and others said it was due to variation in radii, which means the larger the radius the longer the arc. In the end, I tried to problematize them in the case where two aspects, radius and central angle varied at the same time. Students came up with different arc lengths (Teacher B).

Similarly, in his reflective journal, Teacher B wrote: There are many things I have learnt through these rounds of learning studies. It includes how to engage students in critical aspects for them to learn what is taught. I learnt that varying some aspects while keeping invariant others was important for students to focus on a varied aspect(s). One thing I gained in this round is that critical aspects depend on students'" difficulties with what is to be taught. But, it is so challenging to frame them even if you know the students' difficulties (LSCTRJ).

Based on Teacher B's excerpt and reflective Journal, it seems he experienced SCL as *engaging students* to attend to critical aspects of the object of learning. The major focus of the teacher in this learning study was on the organization of critical aspects of what is taught. Although Teacher B experienced SCL as the object of learning, the subject content receded in his background. He used the pedagogy of variation and SCL framework as a means to engage students in discerning the central angle and radius of circular objects separately and simultaneously. However, Teacher B was still sceptical of his ability to identify and structure critical aspects of the object of learning using variation. It sounds like he still required much knowledge of the variation theory.

DISCUSSION

This study found that there is a considerable relationship between teacher's level of understanding of the learning study and their learning. Before the learning study, it was found that the top-down initiative (traditional TPD, such as seminars, and workshops) was used to train teachers on SCL innovation. As such, teachers understood SCL as a particular set of codified instructional methods to adopt. Nevertheless, they failed to practise SCL under the prevailing school challenges. It seems the technical way of understanding SCL did not enable teachers to use their day-to-day practical and reflective experiences in teaching. These experiences could have enabled teachers to modify, renovate or readjust the SCL pedagogical strategies about their school and cultural context (Msonde & Msonde, 2019; Kiely & Davis, 2010; Poekert, 2011). However, the teachers maintained the traditional instruction practices, the result of which they became incapable of practising SCL with the focus on improving student learning. Thus, from the technical perspective view (see also Hargreaves, 2000; Poekert, 2011), it seems that the teachers had technical knowledge of SCL as prescribed by their teacher educators. Nevertheless, the day-to-day practices of SCL were marginalized. As such, their ability to develop new ways of experiencing SCL in their school context was very minimal.

However, it was found that teachers who were engaged in the research lessons 1, 2, and 3, learned how to design and practise SCL collaboratively. They shared their practical experiences with SCL and were allowed to renovate and/or modify it as deemed necessary. Given those opportunities, the teachers changed their way of experiencing SCL from seeing it as *methodological* per se, to subject *content* up to the *object of learning* orientations. Their understanding was gradual and hierarchical. This is because the more they were engaged in learning studies, the better their understanding became focused on student learning.

This study underscores what was found in previous studies that employed learning studies, premised on the variation theory. Previous studies (see Davies & Dunnill, 2008; Keung, 2009; Marton & LO, 2007; Pang, 2006) found that the learning study improved both student learning and teacher professional learning. However, one distinctive feature revealed from this study is the powerful influence of the variation theory in guiding teacher learning. This was particularly, true in designing and enacting SCL lessons focused on student learning of what was taught. The teachers were capable of developing new SCL pedagogical innovations that worked better with their school milieu. Their SCL pedagogical strategies focused on engaging students in attending to critical features of the object of learning‘separately-then-simultenously’ (KI, 2007). Through it, teachers had a new focus and a new way of experiencing and practising SCL. As Kwo (2010) argues “It is not just desirability, but the feasibility of teacher learning is when they break beyond institutional and mental boundaries to claim new focuses and embark on new paths”. The teachers in the learning study group did not intend to develop a new theory, rather, they used the theory in practice to develop new ways of understanding, designing and teaching SCL lessons based on their school's environment. Consequently, it extends a new strand of the theory in teacher's pedagogical understanding and implementation of SCL.

Findings from this study revealed that the variation theory is extremely powerful in the sense that it goes beyond improving student learning of the object of learning as found by previous studies. It also enhances teacher learning an effective way of designing and teaching SCL lessons. This is primarily with the focus on the manner students discern critical aspects of what is taught. This strand is contrary to the previous focus that teachers had on SCL lessons in enhancing student participation during instructions. It was evident that teachers used the theory practically to identify the critical aspects of what was taught. For example, through collaborative meetings, they identified critical aspects in mathematics such as directional, perpendicularity, length, sides' and ratios for improving student learning of trigonometric ratios in research lesson 1. This was because at this stage teachers had a partial understanding of the variation theory. In research lessons 2 and 3, they used students' experiences of the object of learning to identify critical aspects of what was taught. They found vertical and horizontal distances as critical aspects for student learning slope in research lesson 2. Similarly, in lesson 3, they identified the central angle and radius as critical aspects for students learning the arc length. Teachers learned how to create dimensions of variation. That is, they designed lessons in terms of patterns of variation and invariance of critical aspects. They also enacted their lessons focused on engaging learners in discerning critical aspects of the object of learning. It was in terms of what aspect(s) varied and what were kept invariant, even though, they improved gradually.

Thus, we see from this study that teachers in learning studies become, "interdependent innovators, problematizing and reflecting from their practices. Therefore, the variation theory guided teachers to come up with a new alternative pedagogy, which was practical under their prevailing school challenges of large classes, inadequate resources, and teacher overload. Even though the teachers' awareness of the variation theory seemed to improve gradually, there was much to be desired. Data showed that teachers had challenges in identifying and structuring critical aspects of the object of learning. They did not discern angle position, triangle orientation, and angle size aspects in teaching trigonometric ratios in research lesson 1, and used the pedagogy of variation intuitively. They failed to use opportunities within their lessons to enable students to attend to the vertical and horizontal distances simultaneously in experiencing slope in research lesson 2. Moreover, the teachers partially involved students in discerning the central angle and radius simultaneously in experiencing arc length, especially in research lesson 3. These findings imply that the teachers' level of understanding of the variation theory is related to their ability to involve students in attending to critical aspects of the object of learning.

CONCLUSION

This study concludes that teachers engaged through learning studies guided by the variation theory, may improve their understanding of SCL with a focus on student learning of complex mathematics objects of learning. When teachers share their experiences with SCL, they come up with a wider understanding of the best pedagogical implementation of SCL. In doing so, they unfold the SCL teaching challenges, especially in clouded classroom settings. Such understandings and practices transform the teachers' superficial way of experiencing SCL as methodological oriented for enhancing students' participation in classroom transactions to perceiving SCL in a complex manner that focuses on students discerning critical aspects of the object of learning progressively. Moreover, the learning study is powerful as it transcends from improving student learning to teacher learning the pedagogy of SCL with the focus on enabling students to appropriate the object of learning sequentially and simultaneously.

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